Good Afternoon
Please have out your LCM/GCF worksheet...volunteers? have your book opened to p 31

Homework

Math
p 32 letters A, B, C, D
Bell Work Packet all due on Wednesday

Science
N/A

Interims...signed and returned

2.2 Looking at Cicada Cycles: Choosing Common Multiples or Common Factors

Focus Question: How can you find the least common multiple of two or more numbers?

Problem Description
This Problem involves 13-year and 17-year cicadas, which struck a farm in 1998, and asks the question of when they might emerge together again. Students explore strategies for finding common multiples and the least common multiple. Students reason about and predict future occurrences by observing and using patterns in common multiples of numbers. They also determine a lower and upper limit for the least common multiple of numbers by reasoning about the multiplicative structure of the numbers. They use knowledge about factors of a number to determine common factors.

Presenting the Challenge
Tell the class about 13-year and 17-year cicadas, perhaps by reading the introduction to a class. Be sure to allow the students to do the work. If you give too much explanation in the launch, your students will all use the same strategy.

Here is another situation that considers when two or more things will happen again at the same time. Your challenge is to find when the two kinds of cicadas will appear again at the same time.

Did You Know?

Female cicadas lay their eggs in tree branches. When the young cicadas hatch, they drop to the ground and burrow into the soil. They remain underground for 13 or 17 years, feeding off juices from tree roots. Sometimes 13-year cicadas can overlap with 13-year cicadas. This happened in Missouri in 1990.
LOOKING AT CICADA CYCLES

Problem 2.2

Stephan’s grandfather told him about how cicadas sometimes damaged his orchard. One year, there were so many cicadas, they wrecked the backs on all of his young trees. Both the 13-year and the 17-year cicadas came up that year.

(a) Suppose 13-year and 17-year cicadas both appear this year. When will both types of cicadas next appear together? Explain.

(b) Suppose there are 13-year, 14-year, and 16-year cicadas that all appear this year. After how many years will all three types of cicadas appear together again? Explain.

(c) Stephan developed a method to determine the next time two types of cicadas will appear together. He says that if you multiply the cycles together, you get the next time that both types will appear together. Does Stephan’s method work for any pair of cicadas? If so, explain why. If not, provide an example for which Stephan’s method does not work.

(d) For Questions A and B, decide whether the answer is less than, greater than, or equal to the product of the cicada cycles.

Answers

Problem 2.2

A. The multiples of 13 are 13, 26, 39, 52, 65, 78, 91, 104, 117, 130, 143, 156, 169, 182, 195, 208, 221, 234, etc.

The multiples of 17 are 17, 34, 51, 68, 85, 102, 119, 136, 153, 170, 187, 204, 221, 238, etc.

The least common multiple is 221. So the 13-year and 17-year cicadas will next appear together in 221 years.

B. The multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180, 192, 204, 216, 228, 240, 252, 264, 276, 288, 300, 312, 324, 336, etc.

The multiples of 14 are 14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154, 168, 182, 196, 210, 224, 238, 252, 266, 280, 294, 308, 322, 336, etc.

The multiples of 16 are 16, 32, 48, 64, 80, 96, 112, 128, 144, 160, 176, 192, 208, 224, 240, 256, 272, 288, 304, 320, 336, etc.

The least common multiple of 12, 14, and 16 is 336. So the 12-year, 14-year, and 16-year cicadas will all appear together again in 336 years.

C. Stephan’s method only works when the cycles are relatively prime. Sample: 4 and 6 do not work because the product of 4 and 6 is 24, but the cicadas will next come after 12 years.

D. In Question A, the answer is equal to the product of the two cycles. In Question B, the answer is less than the product of the three cycles.
**Did You Know?**

In Question C, you found an example that showed a general method or statement does not always hold true. In mathematics, an example that disproves a statement is called a **counterexample**. If someone claims that a pattern is true for all cases, you only need to find one counterexample to disprove that claim.

\[ 12 \rightarrow 12, 24, \ldots \rightarrow \boxed{324} \]
\[ 14 \rightarrow 14, 28, \ldots \rightarrow \boxed{280} \]
\[ 16 \rightarrow 16, 32, \ldots \rightarrow \]  

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**ACE Assignment Guide**

Applications: 16–29  
Connections: 40–41  
Extensions: 59–61  

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**Corresponding ACE Answers**

**Applications**

16. 1, 2, 3, and 6; the GCF is 6.
17. 1; the GCF is 1.
18. 1, 3, 5, and 15; the GCF is 15.
19. 1; the GCF is 1.
20. 1, 7; the GCF is 7.
21. 1, 5; the GCF is 5.
22. 1, 2; the GCF is 2.
23. 1, 3, 7, 21; the GCF is 21.
24. D  
25. F
26. D

27. a. 2 packages of hot dogs and 3 packages of buns: 1 hot dog and 1 bun
   b. 10 packages of hot dogs and 15 packages of buns: 4 hot dogs and 4 buns
28. 20 members; each member gets 1 cookie and 2 carrot sticks.
   10 members; each member gets 2 cookies and 4 carrot sticks.
   5 members; each member gets 4 cookies and 8 carrot sticks.
   4 members; each member gets 5 cookies and 10 carrot sticks.
   2 members; each member gets 10 cookies and 20 carrot sticks.
   1 member; the member gets all 20 cookies and 40 carrot sticks.

29. a. Answers will vary. Sample: The Morgan family buys a 12-pack of bottled water and a 24-pack of boxes of raisins. Each person in the family gets the same number of bottles of water and the same number of boxes of raisins. How many people could the Morgan family have?
   b. Answers will vary. Sample: John eats an apple once a week. Ruth eats an apple every third day. If they both eat an apple today, when will John and Ruth next eat an apple on the same day?
   c. The Morgan family could have 1, 2, 3, 4, 6, or 12 people; these numbers are common factors of 12 and 24. John and Ruth will next eat an apple on the same day in 21 days; this problem involves overlapping cycles, so it can be solved with common multiples.
Connections
40. $4,995,000$; multiply $4,995$ by $1,000$ for the three factors of $10$.
41.  a. In $2$ hours, the jet will travel $12 \times 2 \times 60 = 1,440$ kilometers. In $6$ hours, the jet will travel $1,440 \times 3 = 4,320$ kilometers.
   b. In $6$ hours, the jet will travel $4,320 - 1,440 = 2,880$ kilometers more than in $2$ hours.
   c. In $4$ hours, the jet would travel twice as many miles as in $2$ hours, or $2,880$ kilometers.

Extensions
59. $90$ years
60.  a. $36$
    b. $0$
    c. Eric forgot that multiplication is commutative, e.g., $3 \times 4 = 4 \times 3$. He only needs to know $28$ different products. Also, zero times anything is zero, and $13$ of the $49$ computations involve zero as a factor. (Note: There are only $19$ different answers possible using two factors from $0$ through $6$.)

61.  a. $12$-year cicadas would meet $2$-year predators either every time they emerge or never. The $13$-year cicadas would encounter predators every other time they emerge, so they could be better or worse off depending on whether the predator came out on odd or even years.
    b. The $12$-year cicadas would meet one or both types of predators every time they emerge. The $13$-year cicadas would meet the $2$-year predators every other time they emerge, and the $3$-year predators every third time they emerge. This means that it would be $6$ cycles, or $78$ years, before the $13$-year cicadas would have to face both predators again. They are better off than the $12$-year cicadas.